

# Computation of Free-Surface Flows Including Particles through Porous Media Structure

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**Abstract.** This study deals with the applicability of the multiphase computational method (MICS), which enables us to calculate the incompressible and immiscible gas and liquid phases including rigid solid bodies modeled by a discrete element method (DEM). To confirm the validity of the computational method, it was applied to the dam-break flows including multiple particles through porous media structure consisting of the particles fixed in the space. The structure is modeled by arranging circular objects in a line with a constant narrow gap between neighboring two objects. It was shown that the calculated results without particles are in good agreement with the experimental results. In addition, it was demonstrated that the movable particles in the flows are trapped by the fixed particles and that the behaviors of fluids and transported particles are reasonably calculated in case that inviscid and viscous fluids are used in the computations.

**Keywords:** Incompressible free-surface flow, fluid-structure interaction, fluid viscosity

## 1. Introduction

In social infrastructure and manufacturing industries, it is one of the indispensable technologies to remove unnecessary substances from various mixtures and take out important target substances. For example, purification of water and elimination of air pollution, which are issues on a global scale in society, are essential environmental and engineering problems. In industries, manufactures such as electronic circuits and graphic printings by inkjet technology require filtration to prevent particles from clogging the nozzles. Removal of unwanted matter in fluid becomes more important from now on in order to achieve high reliability.

Experiments and computations of the liquid phase membrane separation have been carried out [1]. As direct observation of separation process of liquid and solid is difficult, numerical simulation is an effective tool to understand the filtration mechanism. Computational methods according to membrane types have been studied from filtration with large pore sizes to micro-filtration with small pore sizes. Aiming to clarify the mechanism of particle and fluid flows through a porous media structure, we have constructed a simple

2D simulation model by implementing MICS [2] together with DEM (Discrete Element Method) [3] and interface tracking method to detect free-surface profiles on Euler mesh.

## 2. Formulation of mathematical model

Assuming incompressible Newtonian-fluid, the incompressible condition and momentum equation of gas-liquid mixture fluids are given by

$$\frac{\partial u_j}{\partial x_j} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = g_i - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \quad (2)$$

where  $u_i$ ,  $x_i$  and  $g_i$  are components of velocity, Cartesian coordinate and gravitational acceleration respectively.  $\rho$ ,  $t$ ,  $p$  and  $\mu$  are density, time, pressure and viscosity respectively.

To detect the free-surface profiles, the following scalar-convection equation is used:

$$\frac{\partial \phi}{\partial t} + \frac{\partial(\phi u_j)}{\partial x_j} = 0 \quad (3)$$

After solving the above momentum equations in all computational area, the fluid forces acting on the solid bodies are estimated with the volume integral of the pressure-gradient and viscous terms in Eq. (1) as  $f_i$  as follows [2]:

$$f_i = \alpha_k \Delta V \sigma_k \left[ -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left\{ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right\} \right] \quad (4)$$

where  $\alpha_k$ ,  $\Delta V$ ,  $\sigma_k$  are volume fraction of a particle in an element, volume of an element and density of a particle respectively. With the derived  $f_i$  and solid-solid contact forces  $s_i$  by DEM, the translational and rotational motions are calculated by

$$m \frac{\partial u_i}{\partial t} = f_i + s_i + g_i \quad (5)$$

$$I \frac{\partial w}{\partial t} = m_c \quad (6)$$

where  $m$  is mass of particle.  $I$ ,  $w$ , and  $m_c$  are moment of inertia, angular velocity and external torque calculated with  $f_i$  and  $s_i$  respectively.

## 3. Computational methods and results

As shown in Fig. 1 gas and liquid are used to conduct a bench mark test, in which liquid column collapses in the gas. The computational domain is  $W=1$  [m] in width and  $H=1$  [m] in height. The initial size of liquid is  $W_f=0.3$  [m] in width and  $H_f=0.6$  [m] in height. Spherical solids are placed to represent particles or a porous structure: in the former case all the objects are free to move, in the latter case the right-hand four objects are fixed as

a porous structure. The diameter of each solid is 0.1 m and the gap  $W_p$  is 0.05 m. The computational domain is divided into  $100 \times 100$  fluid cells used in a finite volume method.

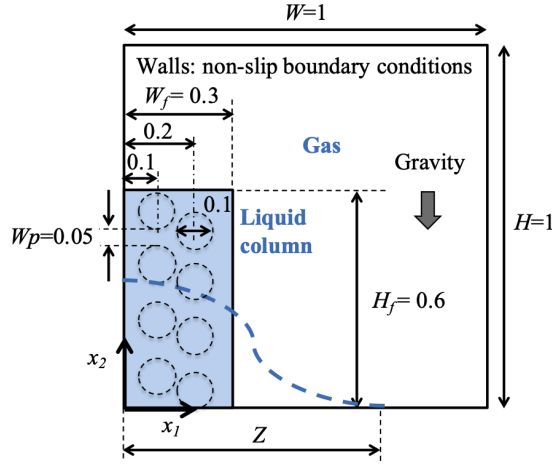


Figure 1: Schematic diagram of computational domain

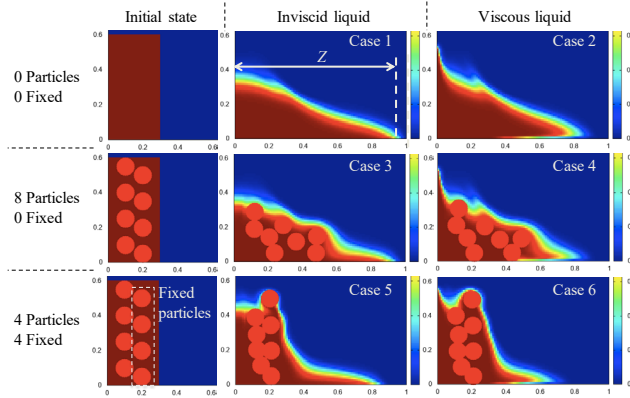


Figure 2: Calculated results with different particle and viscosity conditions

The variables of gas and liquid are expressed with the subscripts  $g$  and  $l$ . The density  $\rho_g$  and  $\rho_l$  are 1.0 and 1000 kg/m<sup>3</sup> respectively. Computations are carried out for six cases: Cases 1 and 2 are inviscid and viscous fluids without particles, Cases 3 and 4 are inviscid and viscous fluids with eight particles, and Cases 5 and 6 are inviscid and viscous fluids with four movable particles and four fixed objects. In the above cases, the kinematic viscosity  $\nu_g$  and  $\nu_l$  are  $1.0 \times 10^{-5}$  and  $1.0 \times 10^{-2}$  m<sup>2</sup>/s respectively, and the kinematic viscosity of the inviscid fluid is set at 0.

Computational fluid flows at  $t=0.3$  [s] for the six cases are shown in Fig. 2. In all cases, the initial liquid columns break and the flows move along the bottom surface due to gravity force. It can be seen that the front speeds of the inviscid fluids faster than any viscous cases with and without movable and fixed particles. In addition, it is shown that the fluids and

movable particles are trapped by the fixed particles and the front speeds are lower than the cases without fixed particles.

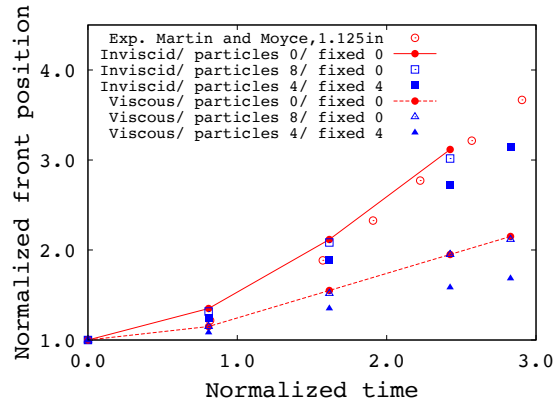


Figure 3: Normalized position  $Z/W_f$  against normalized time  $t\sqrt{2g/W_f}$

Fig. 3 shows the relationship between normalized front positions and time in the six cases. The results in Case 1 can be compared with the bench mark experiment [4]. Leading-edge represented by the length  $Z$  in Fig. 2, is measured against time.

## 4. Conclusions

The applicability of a numerical model to simulate gas-liquid-particle multiphase flows were discussed with some simple 2D computations. The computational method is based on the MICS including DEM and an interface tracking method for free-surface profiles. First, it was confirmed that the calculated results of gas-liquid dam-break flow are in good agreement with the precedent experiments [4]. Second, the computations of dam-break flows were conducted without and with movable and fixed particles in addition that the liquid fluids are inviscid and viscous conditions. The fixed particles correspond to the porous structures that trap the inviscid and viscid fluids as well as the movable particles.

## References

- [1] Y. Mino, S. Sakai, H. Matsuyama: Numerical simulation of filtration process of particle suspension using Lattice Boltzmann method and Discrete Element Method, *Membrane*, 43:6 (2018), 286–291.
- [2] S. Ushijima, S. Yamada, S. Fujioka, I. Nezu: Prediction method (3D MICS) for transpiration of solid bodies in 3D free-surface flows, *JSCE Journal*, 62:1 (2006), 100–110.
- [3] P. A. Cundall, O. D. L. Strack: A discrete numerical model for granular assemblies, *Geotechnique*, 29:1 (1979), 47–65.
- [4] J. C. Martin, W. J. Moyce: An experimental study of the collapse of liquid columns on a rigid horizontal plane, *Philos. Trans. R. London SerA.*, 224 (1952), 312–324.