Fully explicit computational method for thermal interactions between solids and compressible low Mach number gas flows

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Abstract. This paper proposes the new fully explicit computational method for thermal interactions between solid objects and compressible low Mach number gas flows. The proposed method enables us to simultaneously calculate non-isothermal fluid flows and heat conduction in solid objects on the Cartesian grid system since the phase-averaged model is adopted. In addition, the Courant-Friedrichs-Lewy (CFL) condition for compressible fluids is improved by the control method for the speed of sound. The proposed method is applied to two different numerical experiments and its applicability is discussed.

Keywords: Fully explicit scheme, Solid-fluid thermal interaction, Compressible low Mach number flow, Sound speed, Phase-averaged model

1. Introduction

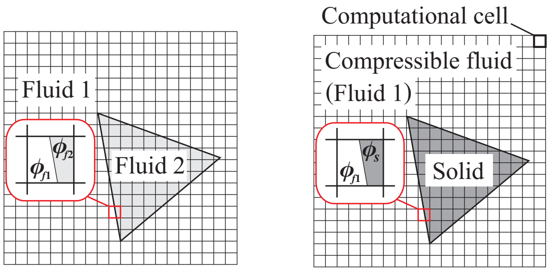
Thermal interactions between solid objects and fluid flows are important phenomena in many engineering subjects. Especially, in some practical problems, large temperature differences occur and the compressibility of fluids is non-negligible even though the flow Mach number is sufficiently small.

On the basis of such background, the computational method for the thermal interactions between solid objects and compresssible fluids was proposed in our previous study [1]. The proposed method enables us to treat complicated shaped and moving solid objects on a simple Cartesian grid system. In addition, the pressure terms in the governing equations for the compressible fluid were treated implicitly to improve the Courant-Friedrichs-Lewy (CFL) condition based on the speed of sound. On the other hand, we proposed the fully explicit scheme for non-isothermal flows by adopting the control method for the speed of sound [2].

In this study, the new fully explicit computational method, which enables us to calculate thermal interactions between solids and compressible low Mach number gas flows, is proposed by combining the above two methods. The proposed method is applied to two different numerical experiments and its applicability is discussed.

2. Numerical method

In this study, a multiphase field consisting of an ideal gas and solid objects (rigid body) is treated as a one-fluid [1] on the Cartesian grid system. To stably calculate mechanical and thermal interactions between solid and fluid phases, which have different physical properties, with simple numerical procedures, we consider two types of multiphase field, namely, multiphase fields A and B, as shown in Fig. 1 [1].



(a) Multiphase field A (b) Multiphase field B

(ϕ is volume fraction of each phase in a computational cell)

Figure 1: Two types of multiphase field considered in computations [1]

In the multiphase field A, we assume that the solid area is full of the fluid. Hereinafter, the ideal gas around the solid object is referred to as a "fluid 1" and solid area is referred to as a "fluid 2". The fluids 1 and 2 are immiscible, while they have same physical properties. The phase-averaged governing equations for the multiphase field A (consisting of fluids 1 and 2) are given by the following mass conservation, momentum, and energy equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_j)}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p'}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \tag{2}$$

$$\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho e u_j)}{\partial x_j} = -p'\frac{\partial u_i}{\partial x_i} + \tau_{ij}\frac{\partial u_i}{\partial x_j} \tag{3}$$

where t is the time, x_i is the component of Cartesian coordinates, ρ is the volume-averaged density, u_i is the mass-averaged velocity, f_i is the external force, and e is the mass-averaged internal energy, respectively. The phase-averaged viscous stress τ_{ij} is calculated from u_i based on our previous study [1].

In Eqs. (2) and (3), p' is the approximated pressure [2] that enables us to control the speed of sound defined as

$$p' \equiv p_0 + \alpha\tilde{p} + (1-\alpha)\bar{P} \tag{4}$$

where p_0 is the initial pressure, α is the constant value to control the speed of sound, which satisfies $0 < \alpha \leq 1$. The pressure fluctuation $\tilde{p}$ is defined as $\tilde{p} \equiv p - p_0$ and p is the pressure that satisfies the equation of state for ideal gas. In addition, $\bar{P}$ in Eq. (4) is the spatially averaged value of $\tilde{p}$. By adopting p' in Eqs. (2) and (3), the CFL condition based on the speed of sound is improved depending on the value of α [2].

As for the multiphase field B, we calculate the phase-averaged heat conduction equation considering physical properties of solid objects as follows:

$$\frac{\partial (\rho C_V)_m T}{\partial t} = \frac{\partial}{\partial x_j}\left(\lambda_m \frac{\partial T}{\partial x_j}\right) \tag{5}$$

where C_V is the specific heat at constant volume, λ is the heat conductivity, and subscript m represents the volume-averaged values between the fluid 1 and the solid object. The above governing equations for the multiphase fields A and B are discretized with the finite volume method on the collocated grid system and solved by the fully explicit method proposed by our previous study [2]. In addition, the mechanical interactions between fluid and solid phases are estimated by the phase-averaging operation for the momentums [1].

3. Results and discussion

3.1. Natural convection in a square cavity containing 16 circular cylinders

The proposed method is applied to the natural convection in a square cavity containing 16 circular cylinders [3, 4] as shown in Fig. 2. The porosity of the cavity is 0.64. The left and right walls are heated and cooled at the constant temperature T_h and T_c, respectively. The temperature difference ΔT between side walls is 1.5 [K] to compare with the reference results [3, 4] obtained by conventional incompressible fluid solvers. The Rayleigh number is 10^5 and Prandtle number is 0.7. Three different values of the heat conductivity ratio λ_s/λ_f are considered, namely $\lambda_s/\lambda_f = 1$, 10, and 100. Here, λ_s and λ_f are heat conductivities of solid and fluid phases. The number of computational cells is 600×600.

Table 1 shows the averaged Nusselt number Nu on the left wall in the steady state. As given by Table 1, predicted Nusselt numbers are in good agreement with reference results. In addition, the maximum value of the Courant number C_a based on the speed of sound is about 36.2 by adopting $\alpha = 5.0 \times 10^{-4}$. The Courant number C_a is calculated as follows:

$$C_a = \max\left\{\frac{|u_1| + A}{\Delta x_1}\Delta t,\ \frac{|u_2| + A}{\Delta x_2}\Delta t\right\} \tag{6}$$

where A, Δt, and Δx_i $(i = 1, 2)$ are the speed of sound, the time increment, and the computational cell size in i-th direction, respectively.

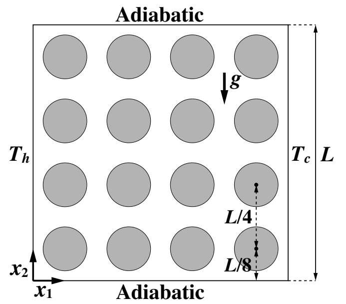


Figure 2: Computational area containing 16 circular cylinders

Table 1: Comparison of Nu in steady state

λ_s/λ_f	Present	Ref. 1 [3]	Ref. 2 [3]	Ref. 3 [4]
1	1.2426	1.2614	1.2986	1.2403
10	2.0398	2.0429	2.0375	2.0153
100	2.4317	2.2957	2.2656	2.3357

3.2. Heat transfer around a rotating gear-shaped solid

Figure 3 shows the computational area. The central circular area of the gear-shaped solid is heated at T_h (= 500 [K]) and all boundaries of the computational area are cooled at T_c (= 300 [K]). The angular velocity ω is π [rad/s]. The fluid is the ideal gas that has same physical properties as the air. The density, the specific heat at constant pressure, and the heat conductivity of the solid object are $\rho_s/\rho_f = 10.0$, $C_{P,s}/C_{P,f} = 0.20$, and $\lambda_s/\lambda_f = 15.0$, respectively. The number of computational cells is 200×200. In addition, α is 1.0×10^{-4} and the predicted maximum value of C_a is about 87.8.

Figure 4 shows the temperature distribution at $t = 3.5$ [s]. The thermal interactions are stably predicted improving the CFL condition based on the speed of sound. In addition, the predicted maximum change rate of the fluid density is about 23.2%. This result shows that the compressibility of the fluid is non-negligible in this numerical experiment.

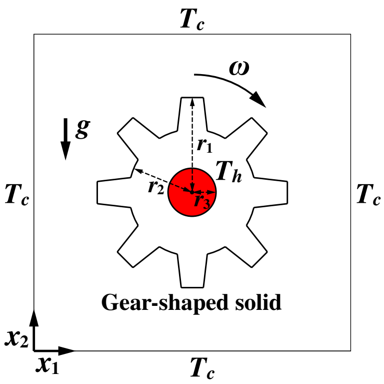


Figure 3: Computational area containing rotating gear-shaped solid

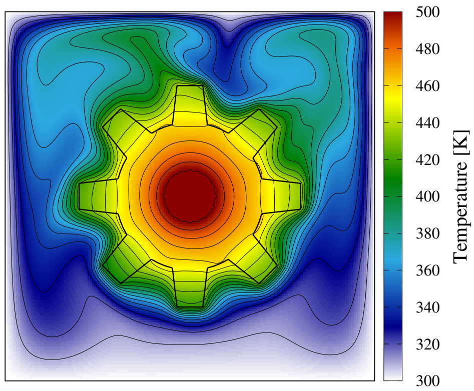


Figure 4: Temperature distribution (t = 3.5 [s])

Acknowledgements

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