Applicability of Pressure-Velocity Correction Algorithm (C-HSMAC method) to Incompressible Fluids with Passive Scalar Convection

S. Ushijima[1*], H. Tanaka[2], W. Liu[2], D. Toriu[1]

[1]ACCMS, Kyoto University, Japan
[2] Faculty of Engineering, Kyoto University, Japan

*ushijima.satoru.3c@kyoto-u.ac.jp

Abstract. In the computations of incompressible fluids, it is essentially important to obtain the velocity and pressure fields that satisfy the incompressible condition ($\nabla \cdot \boldsymbol{u} = 0$) with sufficient accuracy. For this purpose, a pressure-velocity correction method (C-HSMAC method) has been proposed [1] in a finite volume fluid computations. In the C-HSMAC method, three equations including pressure-Poisson equations are iteratively solved until $|\nabla \cdot \boldsymbol{u}| < \epsilon_D$ are satisfied in all computational cells with the given threshold ϵ_D. In this paper, it will be demonstrated that the C-HSMAC method is effective to calculate a simple passive-scalar convection in a two-dimensional cavity flow with an oscillating upper wall.

Keywords: Incompressible fluid, Velocity divergence, Pressure-velocity correction, C-HSMAC method, Passive scalar convection

1. Introduction

The accurate computation that satisfies the incompressible condition, $\nabla \cdot \boldsymbol{u} = 0$, is essentially important in the computations of incompressible fluid, since the numerical errors arising from this condition cause the unphysical change of volume of the fluid [1] as well as other unphysical values of variables which will be shown in this paper. It is noted that such numerical errors from the incompressible conditions are essentially different from the inherent compressibility of the actual fluid properties.

In order to suppress the errors for $\nabla \cdot \boldsymbol{u} = 0$ and control the values $\nabla \cdot \boldsymbol{u}$ explicitly in the numerical procedures, which are not possible in the usual SMAC methods [2], the pressure-velocity correction method, the C-HSMAC method, has been proposed and utilized in the preceding studies [1] [3] [4]. Although the similar computation method that is able to control $\nabla \cdot \boldsymbol{u}$ was proposed as a HSMAC or SOLA method [5] in 1980, it has also been reported that the computational efficiency of the C-HSMAC method is much better than the HSMAC or the SOLA method in our previous paper [6].

In this paper, it will be shown that the convection of passive scalar is affected by the numerical errors arising from the incompressible condition in the finite volume method and that the present C-HSMAC method enables us to obtain the reasonable numerical solutions in the passive scalar problem.

2. Numerical procedures

The governing equations for an isothermal incompressible Newtonian-fluid are given by the following incompressible condition and momentum (NS) equations respectively:

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} \tag{2}$$

with the convection equation of passive scalar c given by

$$\frac{\partial c}{\partial t} + \frac{\partial (c u_j)}{\partial x_j} = 0 \tag{3}$$

where t is time, x_i is the $i-$component of the orthogonal coordinates, u_i is the velocity component in x_i direction, ρ is density, p is pressure and ν is kinematic viscosity. In the following computations, ρ and ν are treated as constants.

The governing equations are discretized with a finite volume method in the collocated grid system [4]. In order to focus on the accuracy of the pressure-velocity correction method, a first-order upwind method was applied to all convection terms to suppress the numerical oscillations. In the pressure computation procedures, the following C-HSMAC method is used, in which pressure-Poisson equations are solved as well as the updates of pressure and velocity components:

do $k = 1, 2, \cdots, k_m$

$$\frac{\partial}{\partial x_j}\left(\frac{1}{\rho}\frac{\partial \phi^k}{\partial x_j}\right) = \frac{1}{\Delta t}\frac{\partial u_{b,i}^k}{\partial x_j} \tag{4}$$

$$p^{k+1} = p^k + \phi^k \tag{5}$$

$$u_{b,i}^{k+1} = u_{b,i}^k - \frac{\Delta t}{\rho}\frac{\partial \phi^k}{\partial x_i} \tag{6}$$

$$\text{exit if} \quad \left|\frac{\partial u_{b,j}^{k+1}}{\partial x_j}\right| < \epsilon_D \ \text{ in all cells} \tag{7}$$

enddo

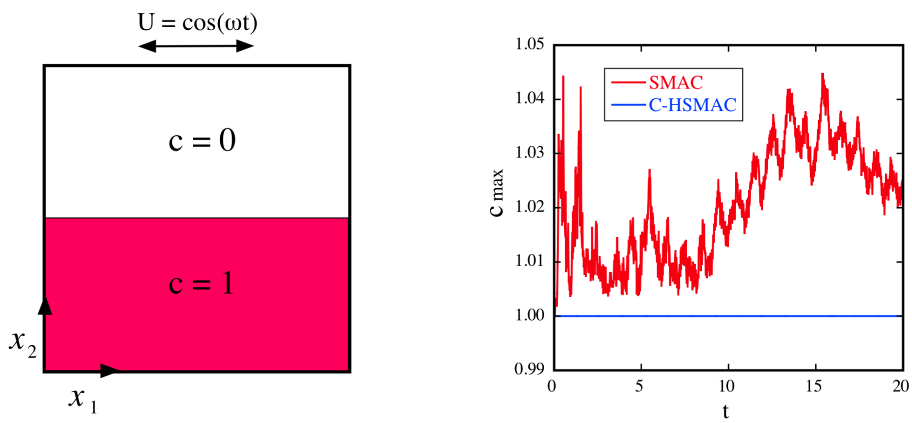
where $\phi = p^{n+1} - p^n$, Δt is time increment, $u_{b,i}$ is cell-boundary velocity which is estimated with spatial interpolation of u_i defined at cell-center point, and ϵ_D is the given threshold.

After the above iterative computations, $|\nabla \cdot \boldsymbol{u}| < \epsilon_D$ are satisfied in all computational cells used in the finite volume method.

3. Results and discussion

Figure 1 (a) shows the initial conditions and square cavity where the lengths are 1.0 both in x_1 and x_2 directions. In the initial conditions, the fluid is static and the scalar c is set 1.0 in $x_2 \leq 0.5$ and 0.0 in $x_2 > 0.5$. The velocity U of the upper wall is given by $U = \cos(\omega t)$ as shown in Fig. 1 (a), where $\omega = 2\pi$.

On the other wall boundaries, non-slip conditions were imposed for velocity, while $\partial p/\partial n = 0$ and $\partial c/\partial n = 0$ are applied to the pressure and scalar on all boundaries. The other conditions are as follows: $\nu = 1.0\times10^{-2}$, $\Delta t = 2.0\times10^{-2}$ and cell numbers are 20×20 in x_1 and x_2 directions. The threshold ϵ_ϕ for pressure-Poisson equation given by Eq.(4) is 1.0×10^{-3} and $\epsilon_D = 1.0\times10^{-10}$. To prevent the numerical oscillation in the higher-order numerical schemes, the upwind method is used for all convection terms.



(a) Computational area and conditions (b) Time history of maximum values of c

Figure 1: Computation of passive scalar c in 2D cavity

Figure 1 (b) shows the time-history of the maximum values of scalar c in the cavity. The SMAC method in Fig. 1 (b) corresponds to the computations with iterative number $k_m = 1$ in the C-HSMAC method. As shown in Fig. 1 (b), the unphysical overshooting values, $c_{max} > 1.0$, can be found, although the stable upwind method is used for all convection terms. Meanwhile, the reasonable values, $c_{max} \approx 1.0$, are maintained in the results of the C-HSMAC method in Fig. 1 (b). The maximum overshooting values are as much as 4% in the case shown in Fig. 1 (b), which are caused by the numerical errors arising from incompressible condition. Therefore, it can be concluded that it is essentially important to solve the incompressible condition as accurately as possible in the computation passive scalar and that the C-HSMAC method is effective for this purpose.

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