Fully explicit computational method for compressible natural convection using reduction technique of pressure propagation

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Abstract.
In this study, we propose a fully explicit computational method for compressible natural convection based on the fractional step method and a reduction technique of the pressure propagation. Since the Courant-Friedrichs-Lewy (CFL) condition based on the speed of sound is improved according to the reduction coefficient, the time increment of the proposed method can be set on the same order as that of a convectional semi-implicit method, which treats pressure terms implicitly. As a result of the application to the natural convection in a square cavity, it is demonstrated that the proposed method enables to conduct computations about 6 ∼ 8 times faster than the conventional semi-implicit method by setting the appropriate reduction coefficient.

Keywords: Fully explicit scheme, Natural convection, Pressure propagation, Fluid compressibility

1. Introduction

In our previous study [1], a fractional step method for non-isothermal compressible gas flows was proposed to calculate thermal interactions between fluids and solid objects under high temperature difference conditions. In our previous method, pressure terms in governing equations written in conservative form are treated implicitly to improve the Courant-Friedrichs-Lewy (CFL) condition based on the speed of sound. As a result, our previous method enables to calculate compressible low Mach number flows using the time increment comparable to that in conventional incompressible flow solvers. However, it is considered that the computational stage of simultaneous linear equations becomes a bottleneck of performance in future large-scale parallel computations. Thus, we propose a new fully explicit scheme for the natural convection with the governing equations of the compressible fluid by using a reduction technique for the pressure propagation.

2. Numerical method

The governing equations of the compressible fluid (ideal gas) are given by following mass conservation, momentum, and energy equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p'}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \tag{2}$$

$$\frac{\partial(\rho C_V T)}{\partial t} + \frac{\partial(\rho C_V T u_j)}{\partial x_j} = -p'\frac{\partial u_i}{\partial x_i} + \tau_{ij}\frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) \tag{3}$$

where t is the time and x_i is the component of orthogonal coordinates, respectively. In addition, ρ is the density, u_i is the velocity component in x_i direction, τ_{ij} is the viscous stress tensor, and f_i is the external force component in x_i direction, respectively. Furthermore, p' is the approximated pressure, λ is the thermal conductivity, T is the temperature, and C_V is the specific heat at constant volume, respectively. The fluid is assumed to be the ideal gas. Thus, τ_{ij} and the equation of state are given by as follows:

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\mu\frac{\partial u_m}{\partial x_m}\delta_{ij} \tag{4}$$

$$p = (\gamma - 1)\rho C_V T \tag{5}$$

where, μ is the coefficient of viscosity, p is the actual pressure, and γ is the specific heat ratio, respectively. On the other hand, p is rewritten as follows with the fluctuation $\tilde{p}$ from the initial pressure:

$$p = P_0 + \tilde{p} = P_0 + \alpha\tilde{p} + (1 - \alpha)\tilde{p} \tag{6}$$

where P_0 is the initial pressure and α is the constant coefficient which satisfies $0 < \alpha < 1$. In this study, we approximate $\tilde{p}$ in the third term on the right hand side of Eq.(6) by $\bar{P}$ which is the spatially averaged value of $\tilde{p}$. As a result of this approximation, p' is defined as $p' \equiv P_0 + \alpha\tilde{p} + (1 - \alpha)\bar{P}$.

By using the above approximation for the pressure, we obtain the propagation equation of $\tilde{p}$ in isentropic flows and in the very short time period from the initial state as follows:

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = \alpha a_0^2 \frac{\partial^2 \tilde{p}}{\partial x_i^2} \tag{7}$$

where a_0 is the speed of sound given by $a_0 = \sqrt{(\gamma p_0)/\rho_0}$ with the initial pressure p_0 and density ρ_0. Equation (7) represents that the speed of sound decreases to $\sqrt{\alpha}a_0$ ($0 < \alpha < 1$). Thus, the larger time increment can be adopted in the proposed method since the CFL condition based on the speed of sound is improved depending on the value of α.

The numerical procedure of the proposed method is divided into three stages [1], advection, diffusion, and acoustic stages, and variables are updated fractionally in each computational stage. In our previous study [1], the pressure terms are treated implicitly in the acoustic stage to improve the CFL condition based on the speed of sound. By contrast, p' is adopted in this study, and the next time step velocity u_i^{n+1} and pressure p'^{n+1} are calculated explicitly as follows:

$$u_i^{n+1} = u_i^{**} + \Delta t\left(-\frac{1}{\rho^{**}}\frac{\partial p'^{**}}{\partial x_i} + f_i\right) \tag{8}$$

$$p'^{n+1} = p'^{**} - \alpha\Delta t(1-\gamma)p^{**}\frac{\partial u_i^{n+1}}{\partial x_i} \tag{9}$$

where Δt is the time increment and the superscript $**$ represents the variable updated in the diffusion stage. By updating u_i^{n+1} and p'^{n+1} as given by Eqs. (8) and (9), the proposed method enables to calculate compressible low Mach number flows efficiently without solving the simultaneous linear equations.

3. Results and discussion

The proposed method is applied to the natural convection in a square cavity using 1 core of the supercomputer system in Kyoto University (CRAY CS400 2820XT, Intel Xeon Broadwell 18cores 2.1GHz x 2 / node). Figure 1 shows the computational area. In this application, the left hand side wall is heated at T_h, and the right hand side wall is cooled at T_c. In addition, adiabatic conditions are imposed on top and bottom walls. The Prandtle number and the specific heat ratio of the fluid are 0.71 and 1.40, respectively. The different Rayleigh numbers Ra are considered in this application, namely $Ra = 10^3$ and 10^6. For $Ra = 10^3$, $\beta\Delta T$ is 3.33×10^{-3} ($<<$ 1). Here, β and ΔT are the coefficient of volume expansion of the fluid and the temperature difference between heated and cooled walls, respectively. By contrast, $\beta\Delta T$ is 0.33 for $Ra = 10^6$ to consider the influence of the fluid compressibility.

The computations are conducted for different values of α with the time increment Δt that derives the maximum Courant number $C_{a,\max}$ based on the speed of sound in the steady state given by Table 1. Here, the Courant number C_a is calculated as follows:

$$C_a = \max\left\{\frac{|u_1| + a}{\Delta x_1}\Delta t,\ \frac{|u_2| + a}{\Delta x_2}\Delta t\right\} \tag{10}$$

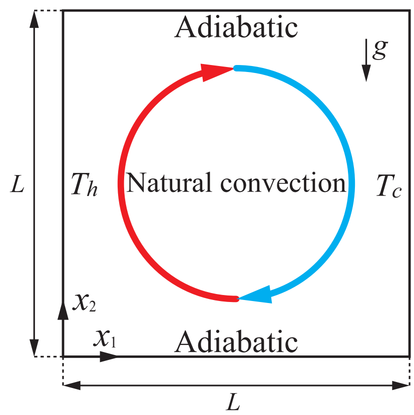

Figure 1: Computational area of natural convection in a square cavity

Table 1: $C_{a,\max}$ in steady state

α	1.00	0.50	0.10	1.00×10^{-3}	5.00×10^{-5}	1.00×10^{-5}
				$Ra = 10^3$		
$C_{a,\max}$	1.06	1.55	3.51	33.4	1.55×10^2	3.34×10^2
				$Ra = 10^6$		
$C_{a,\max}$	1.13	1.57	3.58	35.8	1.57×10^2	–

Table 2: Comparison of S_E/S_I

α	1.00	0.50	0.10	1.00×10^{-3}	5.00×10^{-5}	1.00×10^{-5}
				$Ra = 10^3$		
S_E/S_I	40.4	27.6	12.2	1.28	0.277	0.128
				$Ra = 10^6$		
S_E/S_I	21.4	15.5	6.82	0.680	0.156	–

The calculated Nusselt numbers Nu_h on the heated wall are 1.12 and 8.54 for $Ra = 10^3$ and 10^6 regardless of the values of α. This result shows that α has negligible effects on the accuracy of the computations within the range of values set in this application. In addition, Nu_h are also in good agreement with those reported by the previous numerical study [2] in which the incompressible fluid solver was used. By contrast, some discrepancies occur between our results and reference results [2] in $Ra = 10^6$ since the fluid compressibility is non-negligible in our computational conditions ($\beta\Delta T = 0.33$).

The comparison of the computational elapsed time is given in Table 2. In Table 2, S_E and S_I are elapsed times to calculate by $t' = 1.00$ with the proposed fully explicit method and the conventional semi-implicit method in which the simultaneous linear equations are solved by the Bi-CGSTAB method. The non-dimensional time t' is defined as $t' \equiv tD_T/L^2$. Here, D_T and L are the thermal diffusivity and the length of the square cavity, respectively. In addition, $C_{a,\max}$ in $Ra = 10^3$ and 10^6 obtained by the semi-implicit method are 8.16×10^2 and 8.73×10^2, respectively. As shown in Table 2, S_E is reduced when the smaller α is set in the computations since the larger Δt can be adopted by the reduction technique of the pressure propagation which improves the CFL condition based on the speed of sound. As a result, the proposed fully explicit method enables to calculate the natural convection about $6 \sim 8$ times faster than the semi-implicit one by using appropriate values of α.

References

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