# Implicit Eulerian method for transportation of multiple deformable objects

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**Abstract.** In this study, the implicit Eulerian method was proposed for incompressible flows including deformable solid objects, which are hyperelastic bodies with additional viscosity. The proposed method enables us to calculate the fluid flows and the large deformation of solid objects on the fixed Eulerian computational grid. In addition, the implicit method, which is called C-ISMAC method, is applied to the prediction stage of the MAC method to reduce the computational time in addition to maintain the higher-order accuracy of the schemes. The numerical experiments were conducted for the transportation of multiple deformable objects between parallel two plates with cyclic boundary conditions. As a result, the interactions between fluids and deformable objects were reasonably predicted with different shear modulus $G$. In addition, it was confirmed that the macroscopic viscosity depends on the $G$ of the included multiple bodies.

**Keywords:** Hyperelastic model, Implicit Eulerian method, Macroscopic viscosity

## 1. Introduction

Fluid flows including multiple deformable objects are important problems in various engineering fields and many numerical studies have been conducted so far to predict the characteristics of the flows and deformation of objects [1, 2, 3, 4].

In this study, an implicit method, which is called C-ISMAC method [5], is applied to full Eulerian method for incompressible Navier-Stokes fluids including deformable objects [3, 4], which are hyperelastic bodies with additional viscosity. The C-ISMAC method allows us to reduce the computational time and to maintain the numerical accuracy of the schemes as well. Since the full Eulerian method is employed in this study to solve both of the fluids and solid objects in a fixed Eulerian grid, the proposed method enables us to calculate the interactions between fluids and solid objects at the same time. To confirm the applicability of the proposed method, the numerical experiments are conducted for the Navier-Stokes fluids between two parallel plates including multiple deformable objects. The relationship between the shear modulus and the macroscopic viscosity is discussed through these numerical experiments.

## 2. Numerical methods

The phase-averaged governing equations are derived for the multiphase fields consisting of incompressible Navier-Stokes fluids and visco-hyperelastic solid objects with additional viscosity, which are called 'deformable objects' hereafter. The governing equations are given by the following incompressible condition and momentum equations respectively:

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial(\mu D_{ij})}{\partial x_j} + \frac{\partial(G\phi_s^{\frac{1}{2}} B^{*\prime}_{ij})}{\partial x_j} + \rho f_i \tag{2}$$

where $t$ is time, $x_i$ is the component of Cartesian coordinate system, $\rho$ is density, $p$ is pressure, $\mu$ is the coefficient of viscosity and $G$ is shear modulus. In addition, $u_i$ is the velocity component and $f_i$ is the external force in $x_i$ direction. $D_{ij}$ is the component of the deformation rate tensor and $\phi_s$ is solid volume fraction in a computational cell. $B^{*\prime}_{ij}$ is the deviation tensor of $B^*_{ij}$ given by $\phi_s^{1/2} B_{ij}$ which is the left Cauchy-Green deformation tensor. The equations of $\phi_s$ and $B^*_{ij}$ are given as follows:

$$\frac{\partial \phi_s}{\partial t} + \frac{\partial(\phi_s u_j)}{\partial x_j} = 0 \tag{3}$$

$$\frac{\partial B^*_{ij}}{\partial t} + \frac{\partial(B^*_{ij} u_k)}{\partial x_k} = L_{ik} B^*_{kj} + B^*_{ik} L_{kj} \tag{4}$$

where $D_{ij}$, $B^{*\prime}_{ij}$ and $L_{ij}$ are given as follows:

$$D_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}, \qquad B^{*\prime}_{ij} = B^*{}_{ij} - \frac{1}{3}(\mathrm{tr} B^*{}_{ij})\delta_{ij}, \qquad L_{ij} = \frac{\partial u_i}{\partial x_j} \tag{5}$$

The governing equations are discretized on the collocated grid system and they are solved based on the MAC method [6]. In addition, the C-ISMAC method [5] is used for the calculation of $B^*_{ij}$ and tentative velocity $u_i^*$ in the prediction stage.

## 3. Results and discussion

The proposed method was applied to the Navier-Stokes fluids between two parallel plates including multiple deformable objects. Figure 1 shows the two-dimensional computational area and initial conditions. The radius of circular object is 0.05 and the lengths of the computational area $l_1$ and $l_2$ are 1.0. In addition, the non-slip conditions are imposed on the top and bottom boundaries while the periodic boundary conditions are imposed on the left and right boundaries. The numbers of the computational cells in each direction, $n_1$ and $n_2$ in $x_1$ and $x_2$ directions respectively, are set as $n_1 = n_2 = 280$. The external force $f$ acting in $x_1$ direction is $9.8\times10^{-2}$. It is assumed that the physical properties of the solid objects are equal to those of the fluid phase. Thus, the coefficients of viscosity as well as the densities of both fluids and solid objects are $\mu = 1.0$ and $\rho = 1.0$ respectively. In the numerical experiments, two different values of $G$ are examined; $G = 0.1$ and $G = 0.5$.

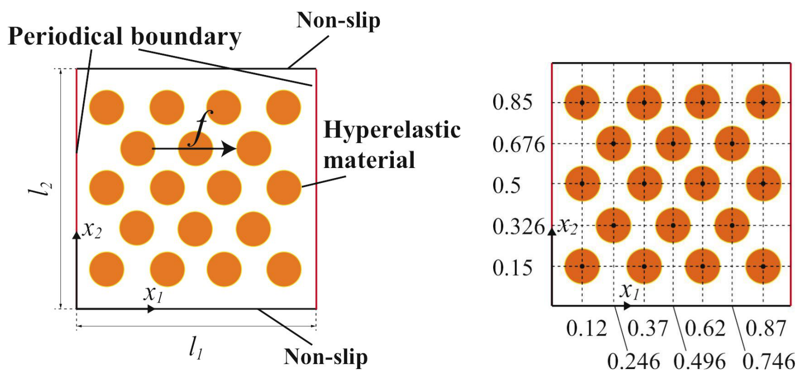
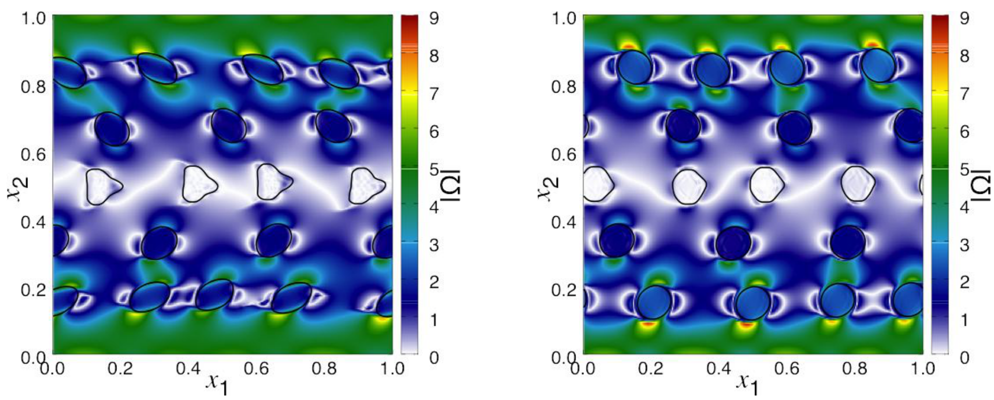



Figure 1: Computational area and boundary conditions

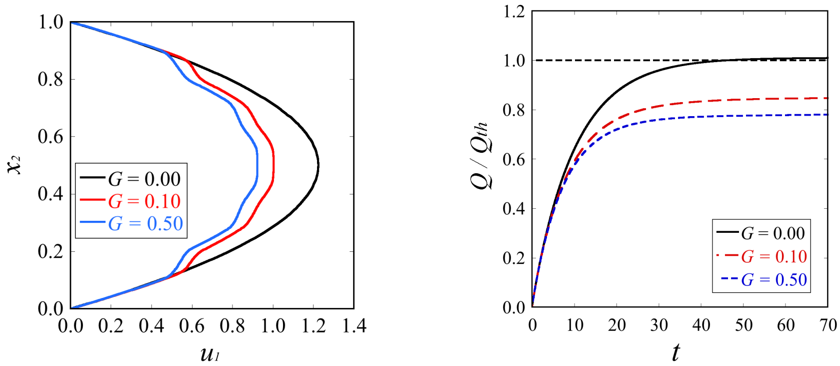
Figure 2 shows the isolines of $\phi_s = 0.5$ that indicate the shapes of solid objects as well as the contour maps of the vorticity component $\Omega$. In Fig. 2, it can be seen that the deformations of solid bodies with $G = 0.1$ are larger than those of $G = 0.5$ and that the vorticity distributions arise around the solid bodies due to the interactions of two phases.




(a) $G = 0.1$ (b) $G = 0.5$

Figure 2: Isolines of $\phi_s = 0.5$ and contour maps of $\Omega$ ($t$=70)

Figure 3 shows the distributions of $u_1$ and the time histories of the normalized flow rate $Q/Q_{th}$, where the $Q$ is the calculated flow rate and $Q_{th}$ is the theoretical value in laminar flow without solid bodies ($G = 0.0$) given by $Q_{th} = fl_2^3/(12\rho\nu_0)$, in which $\nu_0$ is $\nu$ without any solid objects. As shown in Figs.3 (a) and (b), it is confirmed that $u_1$ and $Q$ decrease when the value of $G$ becomes larger. From the above results, the macroscopic viscosity $\nu_m$ is estimated with $\nu_m = f l_2^3/(12\rho Q)$. The values of $\nu_m/\nu_0$ for $G = 0.1$ and $G = 0.5$ are 1.19 and 1.27, respectively. This result shows that the macroscopic viscosity depends on the value of $G$ and that $\nu_m/\nu_0$ becomes large when $G$ increases in the range of the present numerical experiments.




(a) distributions of $u_1$ (b) time histories of $Q/Q_{th}$

Figure 3: Distributions of $u_1$ and time histories of $Q/Q_{th}$

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